

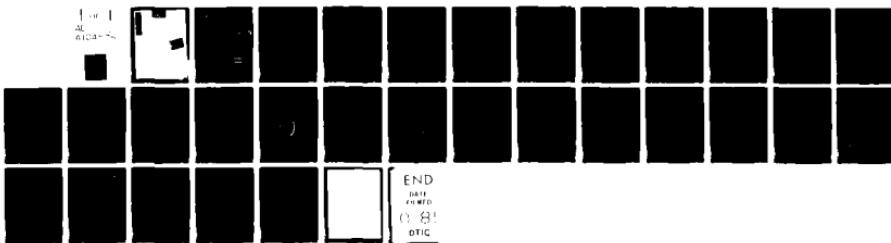
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ANALYTICAL INVESTIGATION OF MOTIONLESS COIL STORAGE (MCS) SYSTEM--ETC(U)  
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AERO MECHANICS DEPARTMENT

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ANALYTICAL INVESTIGATION OF  
MOTIONLESS COIL STORAGE (MCS) SYSTEM

Final Report

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The main problem in the MCS development program is the operational limitation of pay-out speed to 1500 feet/minute associated with the formation of a "reverse loop". This analytical investigation identifies the mechanics of reverse loop initiation, and provides a physical explanation of the phenomena. Then by means of a dynamic analysis, quantitative relations are provided among the variables which control reverse loop formation and operating parameters of the system. Finally, on the basis of quantitative analytical results, a solution is suggested in the form of a design modification required for practical elimination of reverse loop formation and attainment of higher operational pay-out speeds.

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SUMMARY

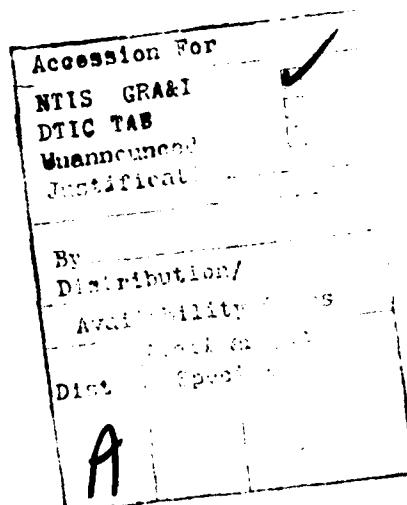
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## I. INTRODUCTION

The motionless coil storage system (MCS) under development at the NAVAIRDEVcen is an advanced cable handling system designed for the rapid pay-out and reel-in of towed antenna (up to 30,000 ft.) for airborne AVL communications. Objectives of the MCS development program are high speed (5,000-10,000 fpm for both reel-in and pay-out) in conjunction with low system weight and short operational response time. The MCS concept which centers on the dormant storage of cable in a stationary tub (as opposed to cable storage on a rotating spool) yields reduced system inertia and allows for the very quick attainment of peak cable handling speed. Preliminary development and evolutionary design progress of the MCS are described in references (a) and (b). Among the various designs which were investigated the inside wrap design (employed in the current apparatus) is considered to be most promising.

As shown schematically in figure 1, the cable is driven between a pair of pinch rollers. During feed-in, the cable passes through a spinner which is rotating about the center line of the tub and is wrapped against the inner tub wall. During reel-out tension is applied to the cable to pull it out through the rotating spinner (which is of course rotating in the direction opposite to feed-in) thus releasing cable from the tub.

Reference (b) reports that this method allowed for sucessful cable storage at speeds of 2000-2500 fpm. Pay-out speeds of 6800 fpm for short periods were attained. The cable however, assumes a "reversed loop" configuration at pay-out speeds above approximately 1500 fpm. The reverse loop configuration precludes successful pay-out in an operational environment thus limiting current operational pay-out speed to 1500 fpm. Operational usage with a reverse loop is not practical due to cable torsion.

## II. DESCRIPTION OF THE PROBLEM

The current MCS apparatus provides for cable pay-out up to a rate of approximately 1500 fpm in an operational mode for which the entire length of unsupported cable (i.e. cable released from the inner wall but not yet entered into the mouth of the rotating spinner) maintains a position forward of the rotating spinner mouth. This configuration is shown in figure 2. At speeds above 1500 fpm some portion

of the unsupported cable falls behind the rotating spinner mouth and assumes the reversed loop configuration schematically shown in figure 3. The above modes of operation are termed the normal mode and reverse loop mode, respectively.

Although pay-out speeds of 6000 fpm have been attained in the reverse loop mode of operation under test conditions with cable relatively free from internal torsion, this reverse loop mode performance could not be achieved in an actual operational environment.

The considerable residual internal torsion of the current TACAMO antenna cable (the cable unwinds under tow tension during deployment and the resultant torsion is largely unrelieved upon retrieval) precipitates kinking of the reverse loop and renders the reverse loop mode of operation impossible.

In recognition of the preceding statements, this analysis is largely addressed to the following areas:

- (1) a phenomenological explanation of the reverse loop formation
- (2) a mathematical description to provide a quantitative relationship with operational parameters
- (3) on the basis of (1) and (2) the formulation of a practical solution

### III. PHENOMENOLOGICAL EXPLANATION OF REVERSE LOOP FORMATION

A review of the MCS apparatus and operation indicates initiation of the reverse loop configuration occurs when the cable feedout speed exceeds the speed with which transverse disturbances propagate along the cable (i.e. the transverse wave speed of the cable). More specifically, the initiation of the reverse loop is caused by the rotating spinner moving at a rate such that it imparts a disturbance to the cable at a speed greater than the transverse wave speed. It is noted that the transverse wave speed depends on the tension in the cable which in turn depends upon the operational parameters of the system.

The essential physical character of this mechanism may be explained for a system consisting of an infinitely long, stationary, flexible string under uniform tension  $T$ . Consider a roller moving to the right with a constant velocity  $V$  which transversely displaces the string some distance " $d$ ".

Figure (4) schematically depicts the string configuration associated with four different roller speeds. For a speed  $V$  much less than the speed of a transverse wave  $c$  (where  $c = \sqrt{\gamma/\mu}$ ; see appendix A) the displacement pattern shown in figure 4-a prevails. As the velocity  $V$  approaches  $c$ , however, the length of displaced string ahead of the moving roller is reduced as shown in figure 4-b. When  $V$  is just equal to  $c$ , the sharp edged configuration portrayed in figure 4-c, can be visualized. Finally as shown in figure 4-d, when  $V$  exceeds  $c$ , (i.e. the string tension is insufficient to maintain a wave velocity equal to the roller velocity), the string cannot respond before it is passed by the roller thereby causing the string to lag behind.

The reverse loop phenomena in the MCS is analogous to this fictitious example. In the case of the MCS the long straight string is actually of course curved around the relatively large diameter of the tub (tub dia.  $\gg$  cable dia.) and the moving roller is replaced by the spinner inlet. The essential physics of the reverse loop initiation and the straight string example are identical.

Having established the physical mechanisms of reverse loop formation and operation, this understanding provides the basis for a dynamic analysis (to be presented in the next section) which will quantize the relation between "reverse loop" formation and system parameters.

#### IV. DYNAMIC ANALYSIS

##### A. Normal Mode of Cable Pay-out

Proceeding from the phenomenologically established relations of the preceding discussion, a dynamic analysis is undertaken to quantitatively relate cable tension (and in turn transverse wave speed of the cable) to the MCS operational parameters. The primary system configuration which is considered is steady state pay-out of cable from the MCS apparatus in the so called normal mode of operation (i.e. before a reverse loop has formed).

Figure 5 shows the basic elements of the system under consideration which include the cable, the spinner and the inner wall of the tub. Consistent with steady state operation, it is assumed that the cable is being reeled out at a constant speed  $V_0$ . The angular speed of the spinner  $\omega$ , is related to the pay-out speed by  $\omega = V_0/r_i$ , where  $r_i$  is the radius of the cable wrap (assumed to be the radius of the inner tub).

An analysis based upon a control volume approach and the conservation of moment-of-momentum is presented in this section. An alternate development based upon force/acceleration principles is given in appendix B. The same results were obtained from both approaches.

The control volume to be considered is taken as shown in figure 5. This control volume encloses the entire spinner and rotates with the spinner. Coordinate system  $x, y, z$  is a moving frame of reference fixed with respect to the spinner. For constant rate of cable pay out the  $x, y, z$  system rotates with constant angular velocity  $\vec{\omega}$  (about an axis through the origin) relative to the inertial reference frame  $XYZ$ . It can be shown (see for instance, ref (c)) that the vector equation governing the rate of change of moment-of-momentum as expressed relative to the rotating control volume takes the form given in equation (1) below.

$$0 = \vec{M}_s - \iint_{c.s.} (\vec{F} \times \vec{V}_{xyz}) (\rho \vec{V}_{xyz} \cdot d\vec{A}) - \frac{d}{dt} \iiint_{c.v.} (\vec{F} \times \vec{V}_{xyz}) \rho dV \quad (1)$$

$$- \iiint_{c.v.} \left\{ \vec{F} \times [2\vec{\omega} \times \vec{V}_{xyz} + \dot{\vec{\omega}} \times \vec{F} + \vec{\omega} \times (\vec{\omega} \times \vec{F})] \right\} \rho dV + \vec{M}_A$$

where  $\vec{\omega}$  is the angular velocity of the spinner

$\dot{\vec{\omega}}$  is the angular acceleration of the spinner

$\vec{M}_s$  = Moment about 0 due to surface forces distributed on the control surface

$\vec{M}_A$  = External applied moment about 0

$\vec{F}$  = Position vector of a cable element in the  $x, y, z$  reference frame

$\vec{V}_{xyz}$  = Velocity of a particle in the  $x, y, z$  reference  
i.e. the relative velocity of a particle with  
respect to the spinner

$\rho$  = Mass density

c.s. = Control surface

c.v. = Control volume

The surface integral represents the efflux of moment-of-momentum through the control surface. The first volume integral represents the rate of increase of moment of momentum inside the control volume, while the second volume integral accounts for inertial forces associated with rotation of the control volume.

It is convenient for the system under consideration to resolve the vector quantities of Eq.(1) in cylindrical coordinates  $\vec{e}_r$ ,  $\vec{e}_\phi$  and  $\vec{e}_z$  as follows.

$$\begin{aligned}\vec{M}_s &= (M_s)_r \vec{e}_r + (M_s)_\phi \vec{e}_\phi + (M_s)_z \vec{e}_z \\ \vec{F} &= r \vec{e}_r + z \vec{e}_z \quad ; \quad \vec{V} = V_r \vec{e}_r + V_\phi \vec{e}_\phi + V_z \vec{e}_z \\ \vec{\omega} &= \omega \vec{e}_z \quad ; \quad \vec{M}_A = M_A \vec{e}_z\end{aligned}$$

Recognizing that under the assumed steady-state operating condition, the angular acceleration of the spinner  $\vec{\omega}$  is zero, the z component of Eq.(1) which is the only component of interest in the current problem takes the form

$$(M_s)_z - \iiint_{c.v.} (2\omega r V_r) \rho dV - \iint_{c.s.} r V_\phi (\vec{V} \cdot d\vec{A}) + \vec{M}_A = 0 \quad (2)$$

Referring to figure 5 it can be seen that the following relationships exist

$$(a) (M_s)_z = (\vec{F}_i \times \vec{r}_i)_z = r_i T_i \cos \psi$$

$$(b) \iint_{c.s.} r V_\phi (\vec{V} \cdot d\vec{A}) = r_i (-V_o \cos \psi) (-\mu V_o) = \mu r_i V_o^2 \cos \psi$$

$$(c) \iiint_{c.v.} (2\omega r V_r) \rho dV = \int_0^{r_i} 2\omega r (-V_o) \mu dr = -\mu \omega r_i^2 V_o = -\mu \frac{r_i^2}{T_i} V_o^2$$

where  $T_i$  is the magnitude of the cable tension at a point immediately outside the mouth of the spinner and  $\mu$  is the mass per unit length of the cable. It is also noted that in the interests of simplicity the evaluation of expression (c) proceeded on the assumption of a sharp 90° bend connection between the radial and circumferential portion of the spinner. Substituting the above relations into equation (2) yields:

$$r_i T_i \cos \psi + \mu \frac{r_i^2}{T_i} V_o^2 - \mu r_i V_o^2 \cos \psi + M_A = 0$$

and solving for the tension gives the expression below:

$$T_i = \mu V_o^2 \left(1 - \frac{r_i}{r_i \cos \psi_o}\right) - \frac{M_A}{r_i \cos \psi_o} \quad (3)$$

Equation (3) shows that the tension  $T_i$  is primarily a function of the mass of the cable  $\mu$ , the pay-out speed  $V_o$ , system geometry, and whatever external moment  $M_A$  may be acting on the system. Expression (3) for the tension can now be used to obtain the transverse wave speed  $c$  of the cable from the following relation (see appendix A)

$$c = \sqrt{\frac{T_i}{\mu}} \quad (4)$$

Additionally recalling the discussion of section III, the necessary condition for pay-out operation at speed  $V_o$  without formation of the reverse loop is that the pay-out speed be less than the transverse wave speed. This provides a condition on the cable tension  $T_i$  which can be expressed as

$$c = \sqrt{\frac{T_i}{\mu}} \geq V_o \quad (5)$$

Combining expressions (3) and (5) yields:

$$\mu V_o^2 \left(1 - \frac{r_i}{r_i \cos \psi_o}\right) - \frac{M_A}{r_i \cos \psi} \geq \mu V_o^2$$

or

$$M_A \leq -\frac{r_i^2}{r_i} \mu V_o^2 \quad \text{i.e.} \quad |M_A| \geq \frac{r_i^2}{r_i} \mu V_o^2 \quad (6)$$

Thus relation (6) provides a condition for the magnitude of the negative or retarding moment (the retarding moment in turn leads to higher cable tension) which must act on the system to avoid reverse loop formation at pay-out speed  $V_o$ . This retarding moment (negative  $M_A$ ) is the externally imposed moment which acts on the spinner in a direction opposite to the direction of spinner rotation. The retarding moment present in the current NCS system consists of resisting moments due to frictional forces and aerodynamic drag. It is relatively small. When the pay-out speed reaches a value which requires a retarding moment

higher than that which is presently associated with the MCS system, the cable assumes the reverse loop configuration.

Numerical values of the cable tension and retarding moments which are required to prevent reverse loop formation at two illustrative pay-out speeds are given in a later section.

#### B. Reverse Loop Mode of Cable Pay-out

Figure (3) depicts the operating configuration for the reverse loop mode of cable pay-out. Employing a control volume approach completely analogous to the preceding development the following result for cable tension at the spinner mouth can be obtained.

$$T_1 = \mu V_o^2 \left( 1 + \frac{r_i}{r_i \cos \psi_o} \right) + \frac{M_A}{r_i \cos \psi_o} \quad (7)$$

Comparison of the reversed loop configuration with the preceding analysis indicates that the moments due to cable tension and influx of cable momentum into the spinner mouth are reversed in direction (sign). This reversal of sign is reflected in expression (7) which indicates a higher cable tension for operation with a reversed loop than for normal mode operation. A retarding moment (negative  $M_A$ ) would in this case act to reduce the cable tension.

Reference (b) observed that a reverse loop developed at pay-out speeds above approximately 1500 ft/min. The total length of the loop seemed to increase somewhat as the pay-out speed was initially increased, but, further increases in speed resulted in no additional increase in the length of the reverse loop. These observations suggest that above a certain pay-out speed the length of the reverse loop is essentially independent of the pay-out speed. A physical explanation of this phenomenon is as follows.

Under high-speed reel-out operation, the predominant forces involved in the loop are  $\vec{T}_1$  (tension at the mouth of the spinner),  $\vec{F}_a$  (force required to accelerate the cable from rest), and  $\vec{F}_f$  (frictional force exerted on the cable by the outer wall). The sum of these three forces should be approximately zero. Since the angle  $\psi_o$  (figure 3) is small and  $M_A$  is also relatively small with respect to the forces listed above, we can assume  $\psi_o = 0$  and  $M_A = 0$ . This leads to the following equation of equilibrium in the circumferential direction.

$$(\vec{T}_1 + \vec{F}_a + \vec{F}_f)_{\vec{e}_\phi} = 0 \quad \text{or}$$

$$\mu V_o^2 \left( 1 + \frac{r_i}{r_i} \right) - \frac{1}{2} \mu V_o^2 \left( 1 + \frac{r_i}{r_i} \right) - c_f \left[ \frac{\mu L \left( 1 + \frac{r_i}{r_i} \right)^2 V_o^2}{r_o} \right] = 0$$

where  $C_f$  = coefficient of sliding friction between the cable and the outer wall

$L$  = Length of the reverse loop in contact with the outer wall of the storage tub.

$r_o$  = radius of outer tub wall

Substituting  $r_o = 4.6$ ,  $r_i = 4.4$  and  $k_i = 4.0$  in the above equation leads to the following results:

$$L \approx \frac{1}{C_f} \text{ (feet)}$$

Assuming a value 0.1 for  $C_f$ , the above equation predicts a length of cable in contact with the wall of 10 feet. This is close to the values observed during previous laboratory operation. It is apparent that for a given system geometry the length of cable in contact with the outer wall is a function of only  $C_f$  (the coefficient of friction) and a reduction in the friction coefficient will increase the length of the loop.

The total length of the reverse loop (variation in the total length derives essentially from the variation in the length of cable in contact with the outer wall) also depends on  $C_f$  and is independent of the pay-out speed  $V$ .

#### C. Reel In Tension

The storage tension developed during reel-in also depends upon the external moment  $M_A$  applied to the spinner. To obtain a desired storage tension  $T_i$ , the external moment which must be provided can be derived from Eq.(2) by reversing the signs of both  $\vec{\omega}$  and  $\vec{V}$ . The following equation results:

$$M_A = r_i \cos \psi_o \left[ \mu V_o^2 \left( 1 - \frac{r_i}{r_i \cos \psi_o} \right) - T_i \right]$$

It is noted that if reel-in and reel-out spinner moments of the same magnitude and direction are applied, then the reel-out tension developed at the spinner mouth will equal the storage tension.

#### D. Tension Measurement

Reference (b) reported cable tension measurements obtained during reverse loop pay-out operation. These measurements were considered to be the cable tension at the exit of the spinner before it enters the floating guide tube. These reported tensions are incorrect because account was not taken of cable forces acting on the guide tube.

These cable forces are caused by momentum changes due to changes in the direction of cable velocity. The readings may, however, be analytically converted to tension. For instance, the actual tension associated with the reported operating condition (6500 ft/min.) is higher than the measured quantity by a factor of approximately 3.

#### V. RESULTS

Based on the dynamic analysis presented in the last section, the solution to the pay-out speed problem is to provide sufficient retarding moment to avoid formation of the reverse loop.

The magnitude of the retarding moment required for various pay-out speeds can be calculated from equation (6). The representative results presented below are based on the geometric dimensions of the current system:

Cable Diameter (in.)	Cable Weight (Lb/Ft)	Pay-out Speed (Ft/Min)	Cable Tension (Lb)	Required Retarding Moment (Lb-Ft)	Retarding Force (Lb)
0.160	0.064	6,000	20	97	22
0.160	0.064	10,000	56	270	62
0.210	0.11	6,000	34	175	40
0.210	0.11	10,000	95	460	105

The retarding "forces" in the above table are listed for reference purposes. They represent forces which would be required to act at the mouth of the spinner in order to produce the required retarding moments.

Since the required retarding moment is proportional to the square of the spinner rotational speed it would be advantageous to implement a device that would generate a retarding moment in the same fashion. However, so long as sufficient retarding moment is applied to the spinner to fulfill the requirements of the desired maximum speed then reverse loop formation can be avoided over the full range of pay-out operation.

#### VI. HIGH SPEED OPERATIONAL CONSIDERATIONS

In view of the increased cable tension requirements for high speed pay-out operation suggested by the preceding analysis the following is presented for consideration.

(1) The relatively high cable tension required for pay-out operation without formation of a reverse loop may be incompatible (undercutting will occur) with the low tension wrap currently employed during the MCS reel-in operation.

(2) The reliability of high speed operation will be improved by the implementation of a lightweight level wind mechanism more refined than the articulated spinner presently used in the MCS apparatus.

(3) Increased tension in the cable would imply more severe wear due to the frictional forces developed by passage of the cable through the spinner. In this case a system of pulleys may be a more attractive design alternative than the spinner tube.

## VII. CONCLUSIONS

1. The current MCS system is limited to an operational pay-out speed of approximately 1500 feet/minute due to the occurrence at this speed of a "reverse loop". Operation with a reverse loop is not practical due to cable torsion.

2. The mechanics of the reverse loop formation have been identified and a phenomenological explanation of reverse loop initiation is presented herein.

3. Reverse loop formation is basically a wave phenomenon which occurs when the cable pay-out speed exceeds a maximum permissible speed equal to the transverse wave speed of the cable. The transverse wave speed of the cable is dependent upon the tension developed in the cable under system operating conditions.

4. Prevention of reverse loop formation requires that the tension in the cable be increased sufficiently to provide a cable transverse wave speed greater than the pay-out speed. This can be accomplished by applying a retarding moment to the spinner.

5. A mathematical analysis has provided quantitative dynamic relations for tension and wave speed as functions of system operating parameters.

6. The retarding moments and cable tensions required to prevent reverse loop formation have been determined for pay-out speeds up to 10,000 feet/minute and are presented herein.

7. The tension measuring system (spring loaded, hinged tube) utilized in the prototype system does not indicate the actual operating tension because of momentum change effects. The indicated tension is low

by a factor of approximately three for the reported test conditions. It is possible to analytically convert the experimental measurements to tension by properly accounting for momentum change.

VIII. RECOMMENDATIONS

1. To prevent reverse loop formation, modify the prototype system to apply the required moment as determined in this study.
2. Experimentally verify the analytical conclusions stated herein by operation of the modified system.
3. For high speed operation at tension values sufficient to prevent reverse loop formation consider the following additional design features:
  - a. Reduction of cable wear and abrasion by utilization of pulleys in lieu of tubing or in conjunction with tubing to guide the cable (alternatively if it is decided to retain the tubing, its shape should be changed to minimize friction and wear ).
  - b. Utilization of a more precise level wind mechanism in lieu of the articulated spinner.
  - c. Reel-in at sufficient tension to prevent undercutting during reel-out at increased tension levels.

IX. REFERENCES

- (a) NADC Report No. AM-6927 "AVLF Cable Handling Development Interim Report" of 16 May 1969
- (b) NADC Report No. AM-7005 "AVLF Cable Handling Development Second Interim Report (June-December 1969)" of 21 September 1970
- (c) I. SHAMES "Mechanics of Fluids" McGraw-Hill, 1962

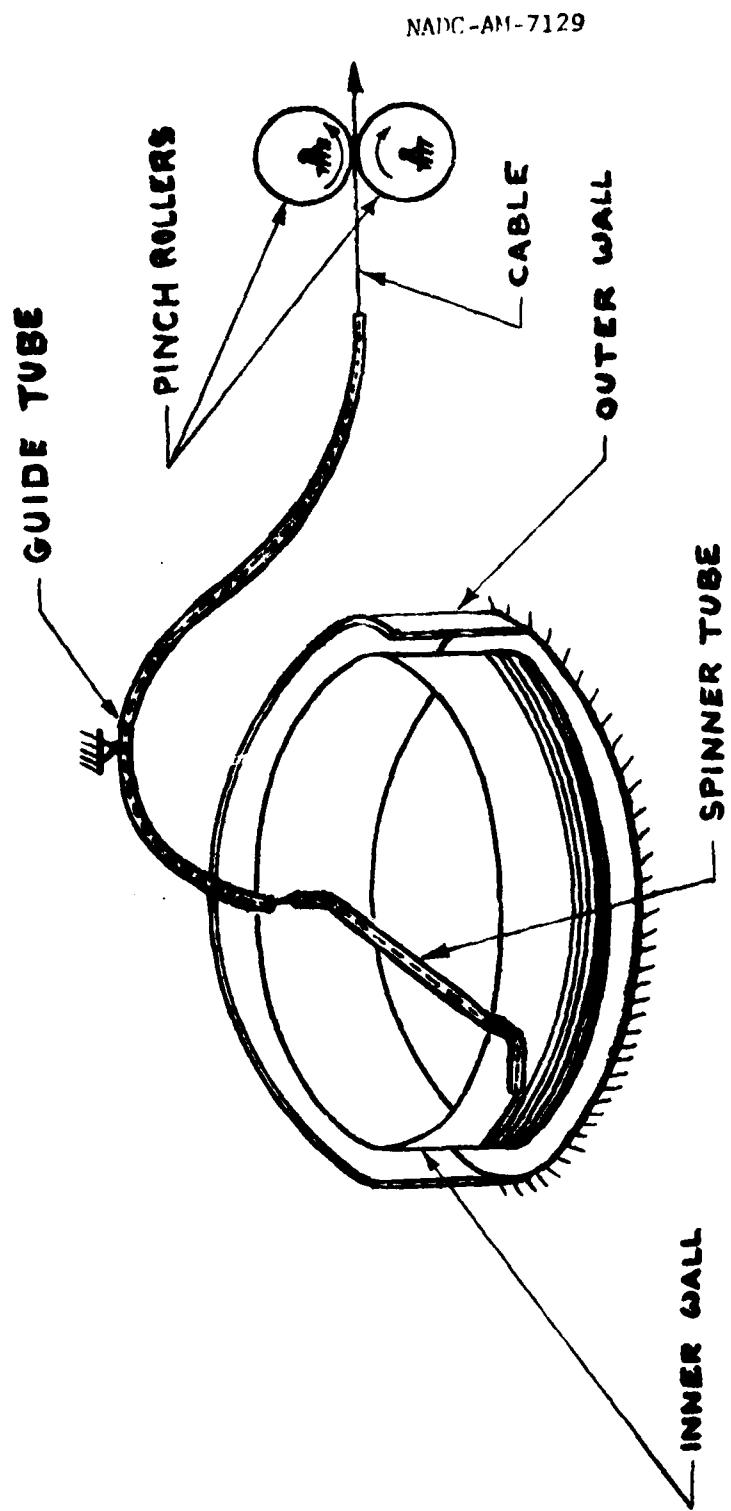


FIG. 1 SCHEMATIC OF MCS

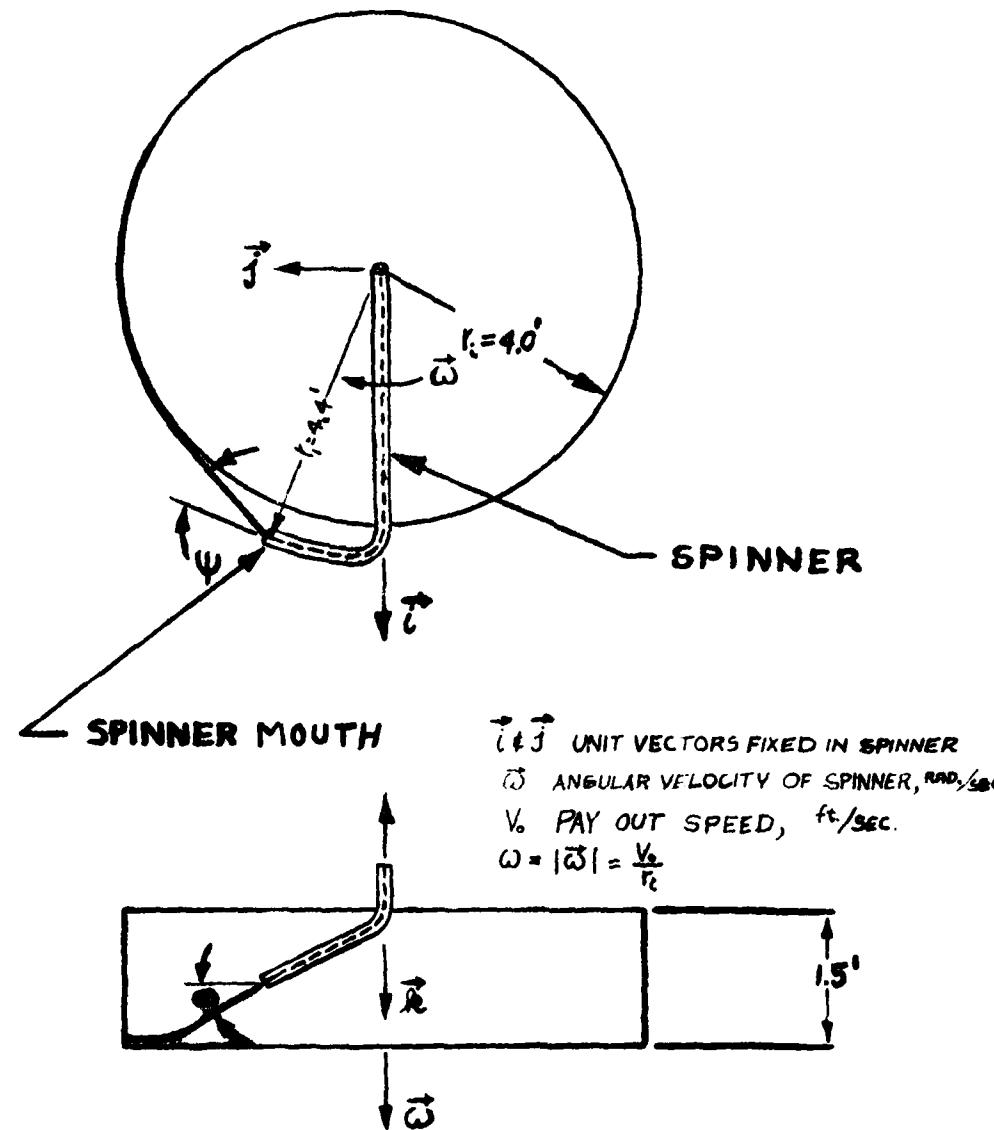


FIG 2 CONFIGURATION WITHOUT REVERSE LOOP

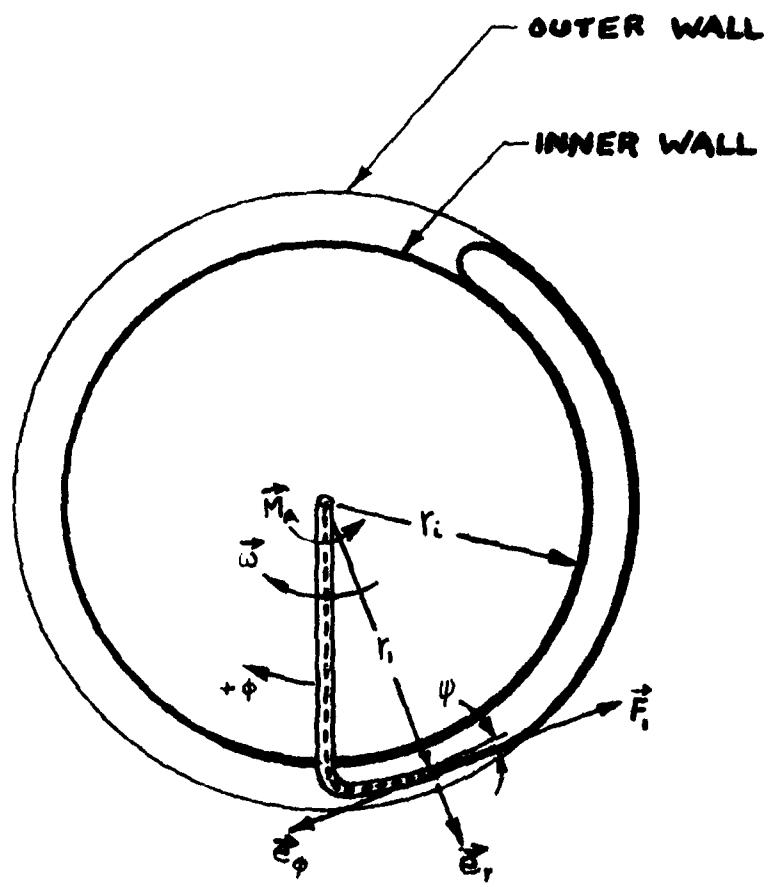
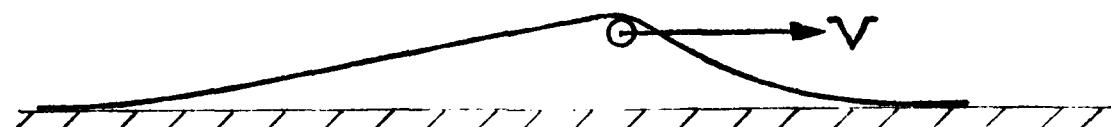


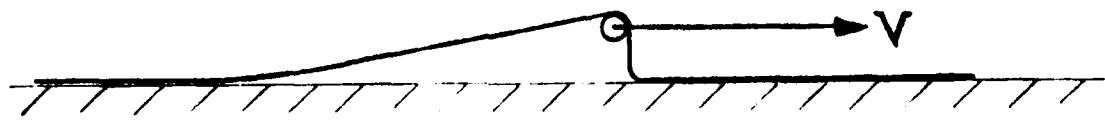
FIG. 3 REVERSE LOOP CONFIGURATION



(a)  $V \ll C$



(b)  $V < C$



(c)  $V = C$



(d)  $V > C$

FIG. 4 MOVING DISTURBANCE APPLIED TO FLEXIBLE STRING

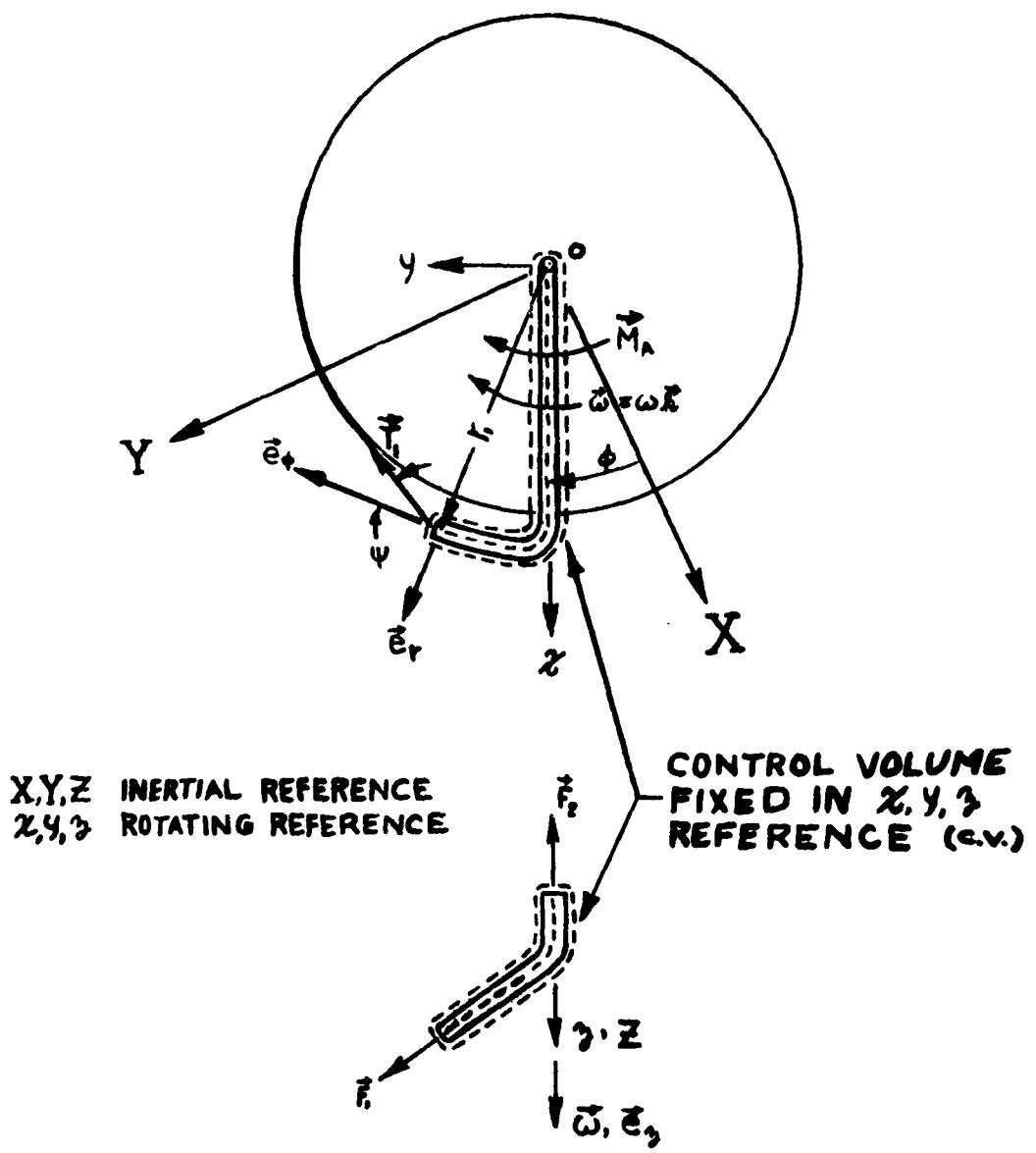


FIG. 5 SYSTEM CONTROL VOLUME

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APPENDIX A  
DERIVATION OF WAVE EQUATION

APPENDIX A

The transverse vibration of a flexible cable with mass per unit length  $\mu$  under constant tension,  $T$ , is described by the following partial differential equation

$$T \frac{\partial^2 Y(x,t)}{\partial x^2} = \mu \frac{\partial^2 Y(x,t)}{\partial t^2}$$

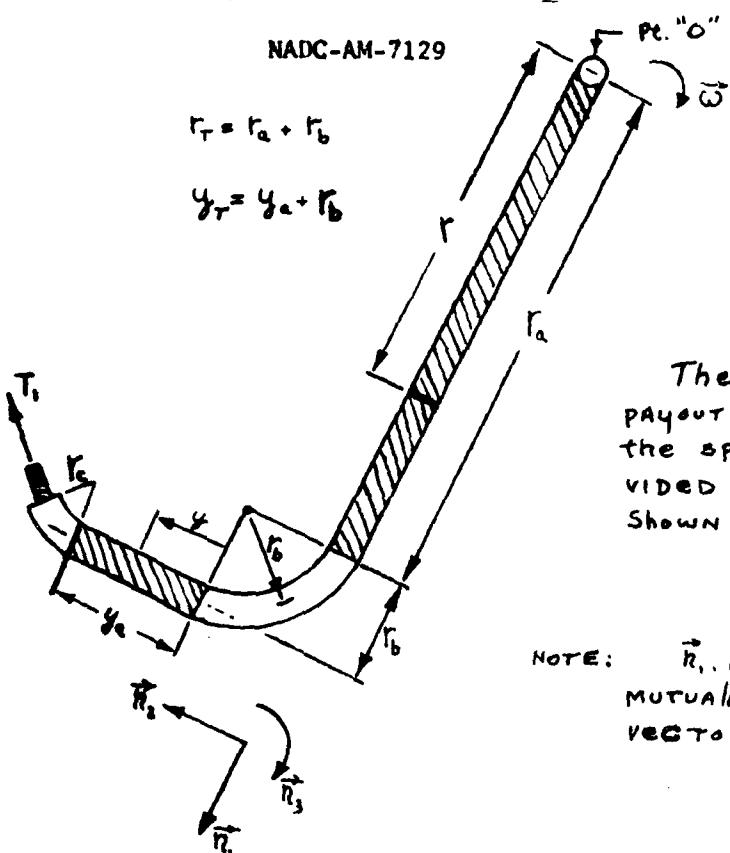
where the transverse displacements,  $Y(x,t)$ , are assumed small. The general solution of this equation is of the form

$$Y(x,t) = f(x-ct) + g(x+ct)$$

where  $f$  and  $g$  are arbitrary functions and  $c = \sqrt{\frac{T}{\mu}}$ . If the solution  $Y(x,t) = f(x-ct)$  is considered as a function of  $x$  for varying values of time  $t$ , it is seen that at  $t = 0$ ,  $Y(x,0) = f(x)$  and at any following time  $t$ ,  $Y(x,t)$  is represented by the same curve  $f(x)$  moved parallel to itself a distance  $ct$  in the positive  $x$  direction. Thus, a solution  $Y(x,t) = f(x-ct)$  represents a curve  $f(x)$  moving in the positive  $x$ -direction with velocity  $c$ . Similarly, a solution  $f(x,t) = g(x+ct)$  represents a curve  $g(x)$  moving in the negative  $x$ -direction with velocity  $c$ . These solutions are generally known as travelling wave solutions and  $c = \sqrt{\frac{T}{\mu}}$  is known to be the transverse wave speed.

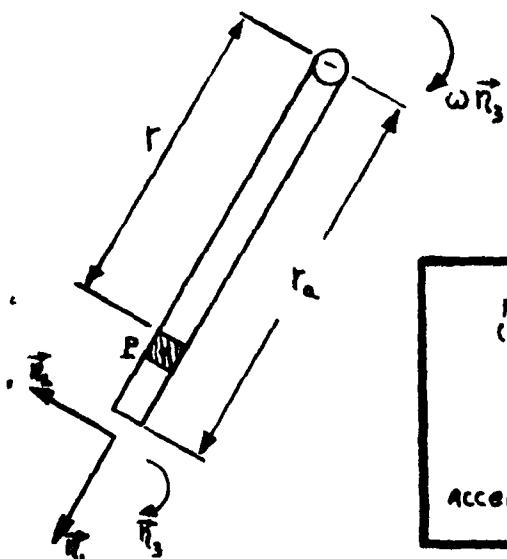
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APPENDIX B  
ANALYSIS OF MCS SPINNER BY  
FORCE/ACCELERATION METHOD

APPENDIX BAnalysis of MCS spinner  
by force/acceleration  
Method

The analysis is concerned with payout operation. For convenience the spinner configuration is divided into the four sections shown to the left.

NOTE:  $\vec{R}_1, \vec{R}_2, \vec{R}_3$  are a triad of mutually perpendicular unit vectors fixed in the spinner.

SECTION 1 : RADIAL PORTION OF SPINNER TUBE

The point P is taken as the center of a portion of cable of length  $dr$  located at some distance  $r$  in the tube.

POSITION VECTOR (relative to "O")	$\vec{P}^E = r \vec{R}_1$
VELOCITY	$\vec{V}^E = \dot{r} \vec{R}_1 + \omega r \vec{R}_2$
ACCELERATION	$\vec{a}^E = (\ddot{r} - \omega^2 r) \vec{R}_1 + (\dot{\omega} r + 2\omega \dot{r}) \vec{R}_2$

CONSIDER the spinner rotating with constant angular velocity  $\omega = V_0/r_c$  where  $V_0$  is the constant rate of cable payout and  $r_c$  is radius of inner tube.

The INERTIA force acting on the incremental length of cable is

$$d\vec{F} = -\mu dr \vec{a}^*$$

AND the incremental moment of this force about the center of rotation is

$$d\vec{M} = (r \vec{r}_i) \times (-\mu \vec{a}^* dr)$$

or

$$d\vec{M} = -(\mu \omega r dr) \vec{r}_i$$

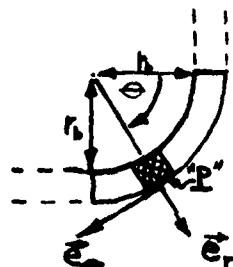
THE TOTAL MOMENT DUE TO THE CABLE IN RADIAL PORTION OF TUBE IS

$$\vec{M}_0 = \int_0^{r_0} d\vec{M} = -\mu \omega r^2 r_i$$

BUT  $\omega = V/r_i$  AND for payout  $r = -y_0$ , so

$$\vec{M}_0 = \mu V^2 \frac{r_i^2}{r_0} \vec{r}_i$$

## SECTION 2: CONNECTING BEND PORTION OF TUBE



POSITION VECTOR (RELATIVE TO "O")

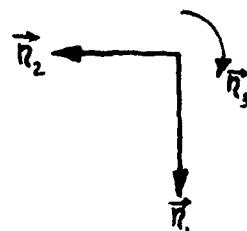
$$\vec{r}^* = r_i \vec{r}_i + r_b \vec{r}_b + r_b \vec{e}_r$$

Velocity

$$\vec{v}^* = r_b (\omega + \dot{\theta}) \vec{e}_\theta + \omega r_b \vec{r}_b - \omega r_b \vec{r}_i$$

Acceleration

$$\vec{a}^* = r_b (\dot{\omega} + \ddot{\theta}) \vec{e}_\theta - r_b (\omega + \dot{\theta})^2 \vec{e}_r - (r_a \omega^2 + r_b \dot{\omega}) \vec{r}_b - (r_b \omega^2 - r_a \dot{\omega}) \vec{r}_i$$



INCREMENTAL INERTIA FORCE

$$d\vec{F} = -\mu \vec{a}^* r_b d\theta$$

INCREMENTAL MOMENT

$$d\vec{M} = \vec{r}^* \times d\vec{F}$$

ASSUMING CONSTANT RATE OF PAYOUT  $\nabla_o$  THE TOTAL MOMENT ABOUT THE CENTER OF ROTATION DUE TO INERTIA FORCES ON THE CABLE IN THE CONNECTING BEND PORTION OF TUBE IS

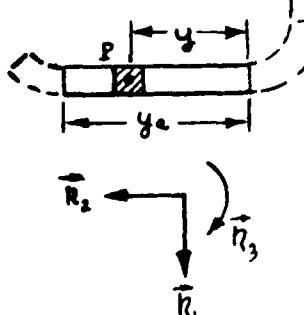
$$M_o = -\mu r_b^3 [(\dot{\theta})^2 + 2\omega\dot{\theta}] \int_0^{\frac{\pi}{2}} [\sin \theta + \frac{r_a}{r_b} \cos \theta] d\theta$$

AND UPON INTEGRATION

$$M_o = -\mu V_o^2 r_a \left(1 + \frac{r_b}{r_a}\right) \vec{R}_3 + 2\mu V_o^2 \frac{r_a^2}{r_b} \left(\frac{r_b^2}{r_a^2} + \frac{r_b}{r_a}\right) \vec{R}_3$$

where  $\omega = V_o/r_a$  AND  $\dot{\theta} = -V_o/r_b$ .

### SECTION 3 : "STRAIGHT" PORTION OF THE TUBE



POSITION VECTOR (RELATIVE TO "O")	$\vec{P}^P = (r_a + r_b) \vec{R}_1 + (r_b + y) \vec{R}_2$
VELOCITY	$\vec{V}^P = -\omega(r_b + y) \vec{R}_1 + (\dot{y} + \omega(r_a + r_b)) \vec{R}_2$
ACCELERATION	$\vec{\alpha}^P = -[\dot{\omega}(r_b + y) + 2\omega\dot{y} + \omega^2(r_a + r_b)] \vec{R}_1 + [\ddot{y} + \dot{\omega}(r_a + r_b) - \omega^2(r_b + y)] \vec{R}_2$

for CONSTANT PAYOUT  $\dot{\omega} = \ddot{y} = 0$

INCREMENTAL INERTIA FORCE  $dF = -\mu \vec{\alpha}^P dy$

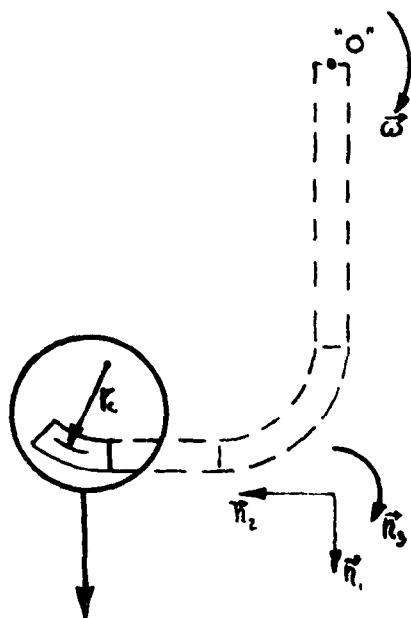
INCREMENTAL MOMENT  $d\vec{M} = \vec{P}^P \times dF = \mu \vec{\alpha}^P dy$

TOTAL MOMENT DUE TO CABLE IN "STRAIGHT" PORTION OF TUBE

$$M_o = \int_0^{y_a} -\mu [2\omega\dot{y}(r_b + y)] dy \quad \text{OR}$$

$$M_o = 2\mu V_o^2 \frac{r_a^2}{r_b} \left[ \frac{r_b y_a}{r_a} + \frac{1}{2} \left( \frac{y_a}{r_a} \right)^2 \right] \vec{R}_3$$

where  $\dot{y} = -V_o$

SECTION 4 : CURVED MOUTH PORTION OF TUBE

POSITION VECTOR (RELATIVE TO "O")	$\vec{r}^E = (r_T - r_c) \vec{r}_1 + y_T \vec{r}_2 + r_c \vec{e}_r$
Velocity, $r_y$	$\vec{V}^E = -\omega y_c \vec{r}_1 + \omega (r_a - r_c) \vec{r}_2 + (\omega + \dot{\psi}) r_c \vec{e}_r$
ACCELERATION (CONSTANT PAYOUT)	$\vec{a}^E = -\omega^2 (r_T - r_c) \vec{r}_1 - \omega^2 y_T \vec{r}_2 - (\omega + \dot{\psi})^2 r_c \vec{e}_r$

$$\text{INCREMENTAL INERTIA FORCE} \quad d\vec{F} = -\mu \vec{a}^E r_c d\psi$$

$$\text{INCREMENTAL MOMENT} \quad d\vec{M} = \vec{p}^E \times d\vec{F}$$

MOMENT ABOUT POINT "O"

$$M_{(1)} = \int_0^{\psi_0} \mu r_i^2 [\dot{\psi}^2 + 2\omega\dot{\psi}] \{ (r_T - r_i) \sin \psi + y_T \cos \psi \} d\psi$$

$$\text{NOTING THAT } \dot{\psi} = -V_0/r_c \quad \text{AND} \quad \omega = V_0/r_i$$

$$M_{(1)} = \mu V_0^2 r_i \left[ 1 - 2 \left( \frac{r_c}{r_i} \right) \right] \left\{ \left( 1 - \frac{r_i}{r_T} \right) \left( 1 - \cos \psi_0 \right) + \left( \frac{y_T}{r_T} \right) \sin \psi_0 \right\}$$

The radius  $r_c$  is considered very small  
compared to  $r_T$  and  $r_i$  so  $M_{(1)}$  reduces to

$$M_{(1)} = \mu V_0^2 r_i \left\{ \left( 1 - \cos \psi_0 \right) - \left( \frac{y_T}{r_T} \right) \sin \psi_0 \right\} \vec{r}_3$$

SUMMING  $\vec{M}_0$ ,  $\vec{M}_1$ ,  $\vec{M}_2$  AND  $\vec{M}_3$  TO OBTAIN THE TOTAL MOMENT ABOUT THE CENTER OF ROTATION DUE TO ALL INERTIAL FORCES AND EQUATING THIS TO THE MOMENT DUE TO THE CABLE TENSION  $T_1$  AT THE SPINNER MOUTH PLUS ANY RESISTANCE MOMENT,  $M_R$  ACTING ON THE SYSTEM. There follows

$$T_1 = \frac{M_R + \mu V_0^2 \left\{ r_T \cos \psi_0 + \left( \frac{y_T}{r_T} \right) \sin \psi_0 - \frac{r_b^2}{r_c} \left[ 1 + \frac{2r_b y_a}{r_a^2} + \left( \frac{y_a}{r_a} \right)^2 + \left( \frac{r_b}{r_a} \right)^2 + \frac{r_b}{r_a} \right] \right\}}{r_T (\cos \psi_0 + \frac{y_T}{r_T} \sin \psi_0)}$$

The above result for TENSION DIFFERS SOMEWHAT FROM THE RESULT DERIVED IN THE BODY OF THE REPORT DUE TO DIFFERENCES IN THE ASSUMED GEOMETRY OF THE SPINNER TUBE. However, for the current case in which  $r_b \ll r_a$  AND  $y_T \ll r_T$  the numerical results are essentially the same. In fact, it can be seen that when  $r_b \ll r_a$  AND  $y_T \ll r_T$  are introduced into the above expression for  $T_1$ , it reduces to

$$T_1 = \frac{M_R}{r_T \cos \psi_0} + \mu V_0^2 \left[ 1 - \frac{r_b}{r_c} \sec \psi_0 \right]$$

This is IDENTICAL TO THE RESULT IN THE BODY OF THE REPORT.

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